**THE CONDENSER PRINCIPLE AND THE EFFECTIVE RESISTANCE ON A NETWORK**

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**Notation used throughout the poster:**

- $\Gamma=(V,E)$ is a network, with $|V|=n$ and $x \mapsto \Sigma_{y \in V}(x,y)$
- $F=\{(x,y) \in E \mid x \in F \land y \in F\}$ for some $x \in F$
- $dF \leftarrow (x,y) \in E \mid x \in F \land y \in E \setminus F \}$ edge boundary
- $\nu$ is the energy of $u$
- $u \in M(F)$ is a positive measure on $F$
- $S(u)|y \in E^{\nu}$ is the support of $\nu$
- $\nu^F$ is the equilibrium measure for $F$
- $\nu_t$ is the equilibrium measure for $F$ using all weights $\nu$.

**Potential Theory**

$L : V \mapsto V \to R$ is a symmetric kernel verifying:
- Energy principle: $L$ is strictly positive definite on $|x \in E^2, \nu|=0$
- Maximum principle: max $x \in E \mapsto L(x)(\nu) \leq \max x \in E(|x \in E, \nu(x)|\nu|y < \nu(x))$

**PROPOSITION 1.**

$L$ verifies the equilibrium principle

$\forall F \subset \subset \subset \subset$, there exists $\nu \in \mathcal{E}(F)$ such that

$L \nu = \nu \in M(F)$

Moreover $S(u)|y \in E^{\nu}$ is the equilibrium measure on $F$.

**Hitting Time**

**Definition:**

The hitting time $H_{x}(x)$ is the expected number of steps for a reversible Markov chain before $y$ is reached when started from state $x$.

**State equation**

$\Delta H_{x}(y) = 1 \text{ if } x \in F$

$H_{x}(y) = 0$

**Poisson equation**

$H_{x}(y) = \frac{1}{2} \sum_{z \in F} \frac{\nu^F(\nu)}{\nu^F} \nu |y \in E |z$

the unique solution such that $H_{x}(y) = 0$

**Explicit expression of $H_{x}(y)$**

$H_{x}(y) = \frac{1}{2} \sum_{z \in F} \frac{\nu^F(\nu)}{\nu^F} \nu |y \in E |z$

**Regular graphs**

$H_{x}(y) = \nu^F(y)$

**SOME EXAMPLES OF HITTING TIMES COMPUTED BY HAND**

**Distance-regular graph**

**Homogeneous tree of degree $k > 1$ and depthness $I$**

**THE EFFECTIVE RESISTANCE**

**Definitions:**

- The effective conductance between $a$ and $b$, $C_{ab}$, is the value of the current from $a$ to $b$ if $a$ and $b$ are set at potential difference $1$

- The effective resistance between $a$ and $b$ is $R_{ab} = C_{ab}^{-1}$

- The escape probability, $P_{out}$, is the probability, starting at a, that a random walk reaches $b$ before returning $a$, i.e., $P_{out} = C_{ab}$

**Equivalent state equation**

$L \nu = \nu \in M(F)$

$\nu |y \in E |\nu|y \leq |\nu|y - \nu(\nu)\nu|y \leq \nu(\nu)$

where $T$ is the Laplacian of $T$ such that $T_{uu} = L_{uu}$ on $F$ and $T_{uu} = \partial_{uu}$ on $D$

$C_{ab} = R_{ab}^{-1}$ and $\nu \mapsto \sum \frac{\nu^F(\nu)}{\nu^F}$

**REFERENCES**


E. Bendito, A. Carmona and A.M. Encinas, Solving Dirichlet and Poisson problems on graphs by means of equilibrium measures, Submitted to Combinatorica.